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Technical Notes

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Damage Index Estimation in Beams and Plates Using Laser Vibrometry

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I. Introduction

MANY structural-health-monitoring techniques developed over the years are based on the detection of changes in the dynamic behavior of the monitored components. Valuable reviews of the state of the art in dynamics-based structural health monitoring can be found in Refs. 1–3. Many studies have investigated the effects of localized and distributed damage on mode shapes, operational deflection shapes (ODS), and corresponding curvatures. The detection of small changes in the deformed configuration of the structure can be used to localize damage and potentially estimate its severity. In particular, small variations from an undamaged state can be highlighted by successive spatial differentiations of the deflections, which are typically required for the estimation of curvature modes and associated strain energy. The evaluation of changes in the curvature of dynamic deflection shapes as a tool for damage detection and localization was investigated in Ref. 4 and subsequently in Ref. 5, among others. In these studies, the dynamic behavior of beams with notch defects and delaminations is studied analytically and experimentally. Their results show the potentials of the technique when applied to the first mode of the beam. The limitation to a single mode was mainly dictated by the limited spatial resolution available in the accelerometers-based experiments. Ho and Ewins[6] formulated a damage index defined as the quotient squared of the corresponding modal curvatures of the undamaged and damaged structure. The damage index was found to be highly susceptible to noise in the measurement data. The measurement errors were amplified because of second derivative computations based on numerical techniques. They also demonstrated that spatially sparse measurements adversely affect the performance of the damage index. Other authors have used curvatures for the evaluation of the strain-energy distribution over the structure under consideration. This approach has been pursued for example by Ref. 7 to formulate a damage index based on the comparison of strain-energy distributions for damaged and undamaged structures. In Ref. 7 and in subsequent papers by the same authors, the technique is applied to beam structures using mode shapes, ODS, or time-domain data to obtain information on both damage location and extent. The same technique has then been extended to plate structures in Ref. 8, where accelerometers are used to measure the deflections to be interpolated for successive differentiation. The results presented in Refs. 7 and 8 show the effectiveness of the technique and are used as a basis for the developments presented in the present study.

A scanning laser Doppler vibrometer (SLDV) is used to measure the dynamic behavior of a test structure. In a SLDV, the laser-beam placement is controlled by a user-defined grid on the structure. This offers the possibility of accurately estimating deflection derivatives of various orders and in turn allows the estimation of curvatures and strain-energy distributions. Such measurement refinement is unattainable in a timely manner using accelerometers and/or strain gauges. Furthermore, the damage index formulation presented in Refs. 7 and 8 requires the use of data from an undamaged structure to be used as a reference. This can represent a problem toward the practical implementation of the technique, as historical data might not always be available, and variations can be induced by a number of reasons other than structural damage. A novel technique is here introduced to allow the generation of baseline information directly from the measured data set. Specifically, reference data are synthesized by undersampling measurements recorded directly on the damaged specimen.

II. Theoretical Background

A. Strain-Energy Ratio

The formulation is introduced with reference to a plate structure, and the corresponding expressions for beams can be obtained as particular cases.

The strain energy for a plate can be expressed as

$$U_i = \frac{1}{2}\int_A D\left[\phi_{i,xx}^2 + \phi_{i,yy}^2 + 2\nu\phi_{i,yy}\phi_{i,xx} + 2(1-\nu)\phi_{i,xy}^2\right]\mathrm{d}A \quad (1)$$

where $\phi_i = \phi_i(x, y)$ defines the plate ODS when excited harmonically at its ith natural frequency. Also, A denotes the plate surface and $D = D(x, y) = Eh^3/12(1-\nu^2)$ is the plate rigidity, with $h = h(x, y)$ denoting the plate thickness, and E and ν are the Young's modulus and the Poisson's ratio of the plate material. The plate is subdivided into an $N \times M$ grid (Fig. 1), so that the strain energy associated with the n, m area can be expressed as

$$U_{i_{nm}} = \frac{1}{2}D_{nm}\int_{A_{nm}}\left[\phi_{i,xx}^2 + \phi_{i,yy}^2 + 2\nu\phi_{i,yy}\phi_{i,xx} + 2(1-\nu)\phi_{i,xy}^2\right]\mathrm{d}A \quad (2)$$

where it is assumed that the plate rigidity is constant over each region. In addition, it is assumed that damage is localized in a single region n, m and that at the damage location

$$U_{i_{nm}}\big/U_i \approx U_{i_{nm}}^*\big/U_i^* \quad (3)$$

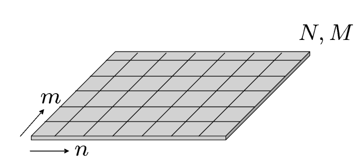

Fig. 1 Schematic of beam and plate divisions.

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so that an estimation of the reduction in stiffness rigidity can be obtained as

$$\frac{D^*_{nm}}{D_{nm}} \approx \frac{U^*_{nm}}{U_{nm}} \times \frac{\int_{A_{nm}} \left[\phi^2_{i,xx} + \phi^2_{i,yy} + 2\nu\phi_{i,yy}\phi_{i,xx} + 2(1-\nu)\phi^2_{i,xy}\right] \mathrm{d}A}{\int_{A_{nm}} \left[\phi^{*2}_{i,xx} + \phi^{*2}_{i,yy} + 2\nu\phi^*_{i,yy}\phi^*_{i,xx} + 2(1-\nu)\phi^{*2}_{i,xy}\right] \mathrm{d}A} = f_{i_{nm}} \quad (4)$$

where D^*_{nm} and ϕ^*_i are, respectively, the rigidity and the ODS of the undamaged plate, and $f_{i_{nm}}$ is the modal strain-energy ratio (SER) for the n, m region. The SER potentially provides indications of both damage location, as well as of its extent. Specifically, $f_{i_{nm}}$ is expected to be equal to one over undamaged regions, and different than one over a damaged region. It is well known how damage affects the strain-energy distribution when it is located near regions of maximum strain-energy. Hence, the visibility of damage is affected by the excitation frequency. For this reason, it is convenient to combine information obtained from the analysis of several modes I and to consider a cumulative strain-energy ratio for region n, m

$$f_{nm} = \frac{1}{I}\sum_{i=1}^{I} f_{i_{nm}} \quad (5)$$

This cumulative index combines the results from several ODS and associated strain-energy distributions. ODS not affected by damage because its particular location will give unit contributions, whereas the index for modes altered by the defect will be combined to provide a robust defect indication.

B. Spline Interpolation of Operational Deflection Shapes

The SER provides a comparison between damaged and undamaged strain energy over the considered region of the structure. In practice, however, it might be difficult to have or obtain baseline information from structures to be analyzed. This in fact assumes that either an undamaged specimen, or that historical data of the same kind as those currently being collected, are available. For this reason, we propose a technique which allows synthesizing undamaged information directly from the measured response. In the proposed approach, the ODS are measured at several locations over the structure, so that spatial derivatives required for the evaluation of SER can be accurately estimated. This is done by performing the spline interpolation of the measured data. Based on the procedure outlined in Ref. 9, the ODS $\phi(x, y)$ for a plate structure can be approximated as

$$\phi(x, y) \cong \sum_{p,q} h_p(x) h_q(y) \Phi_{p,q} \quad (6)$$

where $\Phi_{p,q}$ defines the value of the ODS at location p, q over the plate and $h_p(x)$, $h_q(y)$ are spline basis function.[10] The value of $\Phi_{p,q}$ is the quantity measured experimentally at the sensor location or at a point of a SLDV grid. The spline interpolation of the plate deflection can be used for the prediction of the derivatives of the deflection. In particular, curvature estimations can be obtained by taking derivatives of the spline functions, while keeping the nodal or measured values as weighting parameters. The presence of noise on these parameters hence does not significantly affect the estimation of curvatures. This concept can be described in a concise manner by expressing the plate curvatures as

$$\phi_{,xy}(x, y) \cong \sum_{p,q} h_{p,x}(x) h_{q,y}(y) \Phi_{p,q} \quad (7)$$

This process reduces the numerical errors associated with the presence of noise in the measurements, which is unavoidable, and makes the experimental evaluation of SER feasible. The splines used for the interpolation are quintic splines, which are sufficient to capture the plate deflections and the discontinuities caused by damage. The considered splines, however, do not guarantee continuity and compatibility at the boundaries. This can affect the accuracy of the estimations, particularly when damage occurs near the boundaries of the structure. In the cases considered in this work, such problems are avoided by neglecting the interpolated deflection near the boundary nodes. A more robust treatment of boundary regions is, however, needed and is currently under investigation.

The preceding interpolation procedure offers the opportunity of calculating SER, without the need for baseline data. The availability of measurements on a supposedly undamaged structure to be directly compared with those on a damaged structure is an assumption upon which many damage detection techniques rely heavily.[2,7] Here, we propose a technique whereby damage or anomalies can be obtained from a single measurement. Baseline information is generated by using a subset of the measurement points. The baseline interpolated deflection can be expressed as

$$\phi^*(x, y) \cong \sum_{r,s} h_r(x) h_s(y) \Phi_{r,s} \quad (8)$$

where r, s are a subset of the measurement grid points p, q, such that $r < p$, $s < q$. The resulting undersampling of the data has the purpose of intentionally "missing" any discontinuity or anomaly corresponding to damage, which can generally be detected only through a refined measurement grid. The baseline information can be then differentiated and used for the estimation of the strain energy generically denoted as U^*.

III. Numerical Results

The procedure just presented has been first tested on numerically simulated results for a beam structure. The beam is simply supported at both ends and loaded by a harmonic load of varying frequency. The behavior of the beam is predicted through a finite element (FE) model, formulated using classical Euler–Bernoulli beam elements. The length of the beam is discretized using 80 elements. This large number of elements is selected to replicate experimental measurement density available with SLDV. The nodal displacements corresponding to a harmonic load at a specified frequency are computed and used for the calculation of beam curvature and SER computation. A subset of the nodal displacements is used to generate reference data. The subset is defined by undersampling the available nodal displacements, according to a so-called "decimation factor," which is defined as

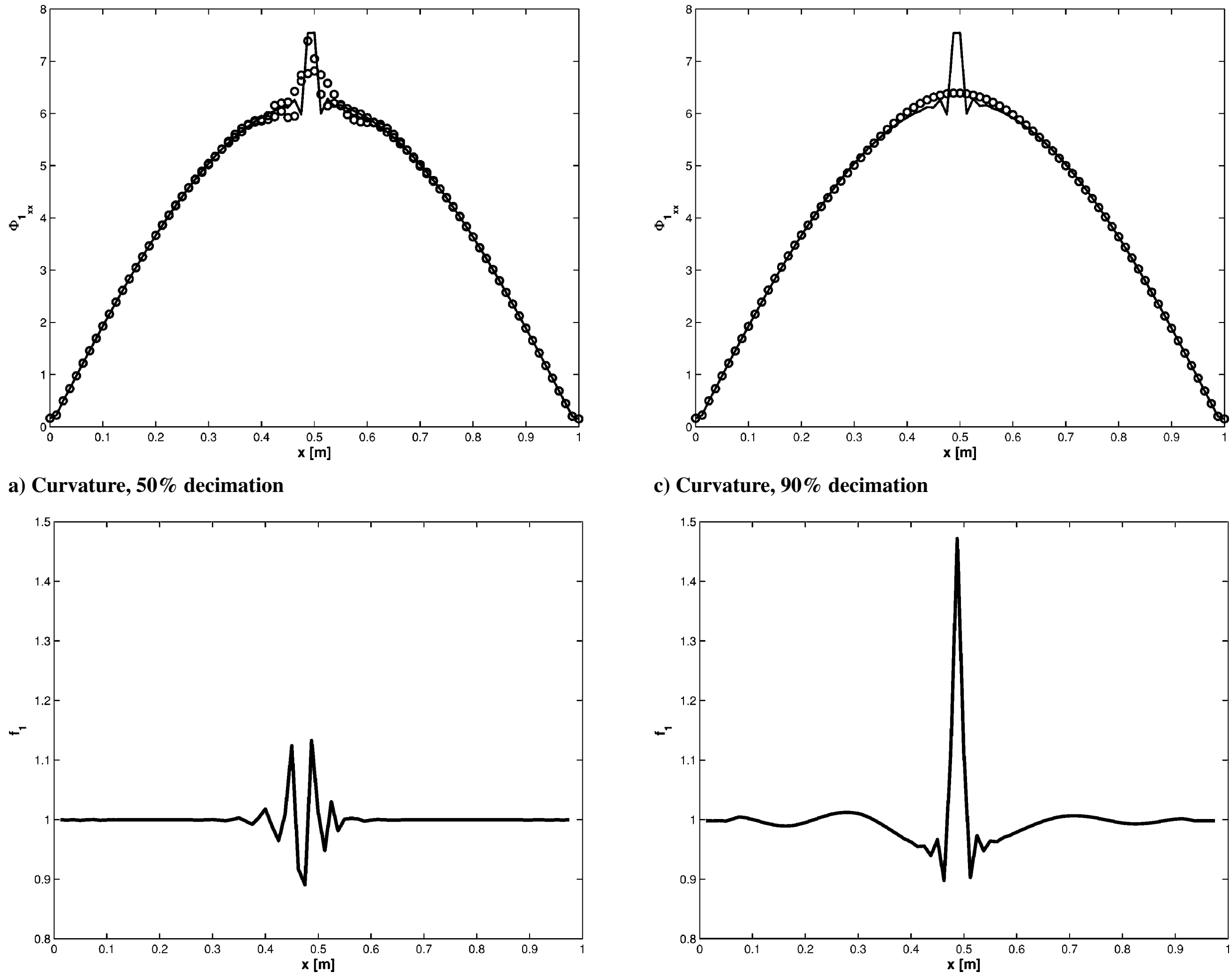
$$d = 100 \times (1 - N_i/N_t) \quad (9)$$

where N_i is the number of nodal displacements used for the interpolation and N_t is the total number of nodal displacements available from the analysis.

Damage in the beam is simulated as a thickness reduction occurring over a single element of the FE mesh. The thickness of the damaged element is denoted as h, and h_0 denotes the thickness of the undamaged beam. Examples of results for an excitation at the first natural frequency of the beam are shown in Fig. 2. The plots, obtained for damage $h/h_0 = 0.9$ at midlength of the beam, show how the first curvature mode clearly highlights the presence of damage. They also illustrate how the selection of the decimation parameter is critical to the formulation of the SER. For low decimations ($d = 50\%$), the synthesized baseline data, indicated as o in the plots, replicate very closely the interpolation obtained from the full data set. The resulting SER estimation highlights some deviation from the unit value at midlength, but does not give a clear indication of the location of damage. The predictions can be significantly improved by increasing the decimation number, which allows synthesizing baseline information that resembles very closely the undamaged configuration. The proper selection of the decimation parameters provides a good estimation for the SER, which, in principle, can be used to assess both damage location and extent. Figure 3 shows how the amplitude of the deviation from unity is in fact proportional to the damage extent. The plots correspond to the cumulative SER obtained from combination of the first 10 modal SERs for both single and multiple damage locations.

IV. Experimental Results

The approach presented in the preceding section is validated experimentally on beam and plate structures. A simple fiberglass beam was first selected as a stepping stone for application to two-dimensional plate specimens.



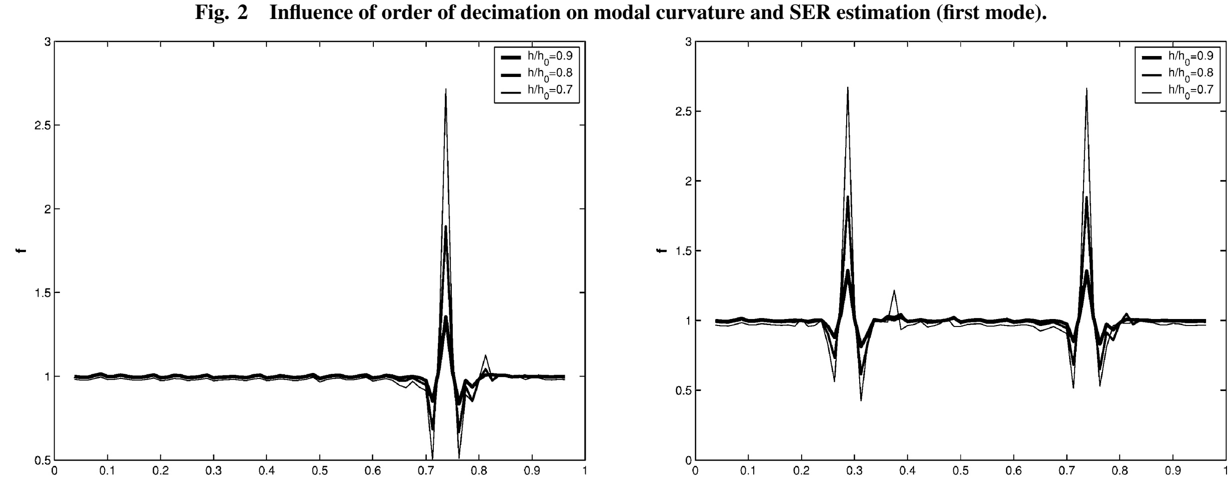
a) Curvature, 50% decimation

c) Curvature, 90% decimation

b) Modal SER

d) Modal SER

Fig. 2 Influence of order of decimation on modal curvature and SER estimation (first mode).



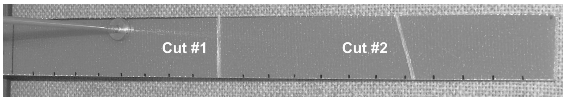
a) Damage at 75% length

b) Damage at 75 and 30% length

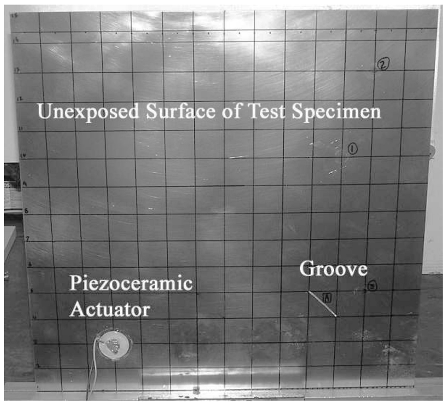
Fig. 3 Cumulative SER for different damaged configurations.

The beams used in the tests are 3.81 cm (1.5 in.) wide and 4 mm (0.155 in.) thick and are cut from a plate fabricated with 12 layers of woven cloth with 0- and 90-deg fiber orientation. The matrix is a polyester resin type. The damaged fiberglass beam used in the tests is shown in Fig. 4a. Damage is inflicted by cutting rectangular grooves using a hand saw. The first considered cut (no. 1) is applied normally to the length of the beam. A second cut (no. 2) is inclined with respect to the beam axis to ensure that the effectiveness of the technique is not affected by a particular orientation of the damage. The depth of each cut is kept at approximately 1.3 mm (0.05 in.), with tolerance requirements that are not strict, as the objective at this time is to simply detect the presence of the damage in the test specimen. The widths of cuts 1 and 2 are, respectively, 1.3 mm (0.035 in.) and 1.3 mm (0.05 in.). The beam is excited by an electrodynamic shaker (Ling Dynamics, Model 102A) over the 0–1000-Hz frequency range. The scanned surface of the beam is covered with a reflective tape to improve the low reflectivity of the considered composite material.

Tests are also carried out on an aluminum plate measuring $35.6 \times 35.6 \times 0.1$ cm ($14 \times 14 \times 0.04$ in.) and cantilevered at the base. A piezoceramic disk of 1.1-in. diam and 0.030-in. thickness is bonded to the plate and used as an exciter. Plate, with actuator



a) Damaged beam



b) Damaged plate

Fig. 4 Experimental setup for a) beam and b) plate tests.

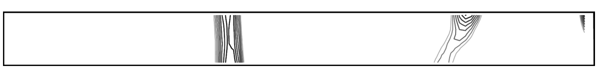
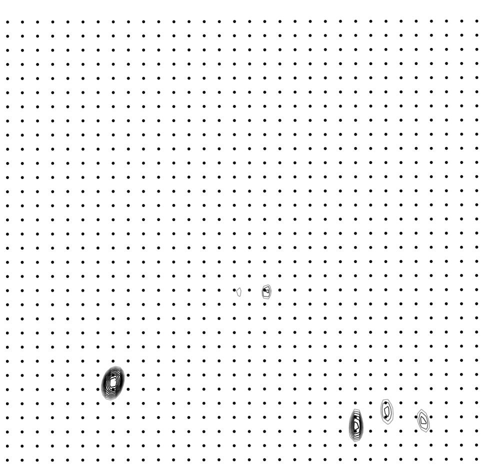
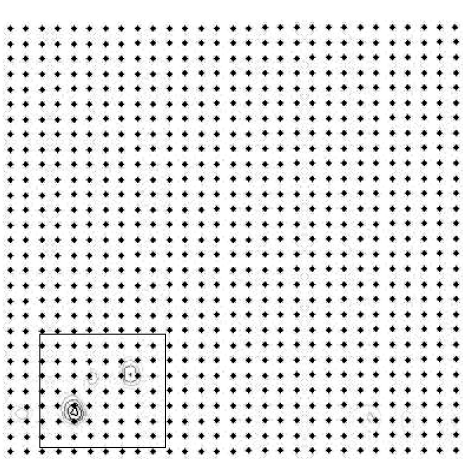


Fig. 5 Cumulative SER estimated using first five modes of beam with two damages.



a) Modal SER (fifth mode)



b) Cumulative SER (first five modes)

Fig. 6 Curvature and modal SER at 293 Hz.

and damage location, is shown in Fig. 4b. The damage is a 3.58-cm (1.41-in.) long, 1.3-mm (0.05-in.) wide, and 0.015 deep groove, which was cut in the plate at the location shown in Fig. 4b. Contrary to the beam case, no surface treatment is needed on the scanned surface of the plate.

The results of SER evaluation for the composite beam are presented in Fig. 5. The SER contours are evaluated by using data from the complete measurement grid and by superimposing the contributions of the five modes of the beam that contribute to the response in the 0–1000-Hz frequency range. Both damages are correctly identified in terms of their location as well as orientation. The advantage of this approach is that no expert intervention is required for the selection of the mode to be analyzed, which can facilitate the process of automating the technique.

Figure 6a shows the modal SER corresponding to an excitation at its third mode. The presented contour plot clearly highlights the location of damage in the plate. Some deviation from unity can be observed at other locations, one of them being located close to the exciter, and the other most likely being caused by measurement and/or numerical noise. This spurious information can be effectively filtered out by considering a cumulative SER obtained from the superposition of several modes. The cumulative SER obtained from the superposition of the first five modes of the plate is shown in Fig. 6b, which provides an unambiguous indication about damage presence and location.

V. Conclusions

An innovative damage index formulation is presented. The proposed technique does not require any baseline knowledge of the structure's behavior before damage occurs. Baseline data are synthesized by using a decimation process whereby data are purposely undersampled in order to miss discontinuities caused by damage. This approach is successfully demonstrated on a composite beam with multiple damages and on a metal plate. In all cases, the damage location was correctly identified. The implementation of the proposed technique relies on the use of a laser Doppler vibrometer, which allows high measurement refinement.

Acknowledgments

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